

Gravitational renormalization of quantum field theory: a “conservative” approach

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Abstract. We propose general guidelines in order to incorporate the geometrical description of gravity in quantum field theory and address the problem of UV divergences non-perturbatively. In our approach, each virtual particle in a Feynman graph should be described by a modified propagator and move in the space-time generated by the other particles in the same graph according to Einstein’s (semiclassical) equations.

1. Introduction

Pauli, long ago [1], suggested that gravity could act as a regulator for the ultraviolet (UV) divergences that plague quantum field theory (QFT) by providing a natural cut-off at the Planck scale. Later on, classical divergences in the self-mass of point-like particles were indeed shown to be cured by gravity [2], and the general idea has since then resurfaced the literature many times (see, *e.g.*, Refs. [3, 4, 5, 6, 7, 8]). In spite of that, Pauli’s ambition has never been fulfilled.

As it happens, QFT is successfully used to describe particle physics in flat [9] (or curved but still fixed [10]) space-time where standard renormalization techniques allow one to obtain testable results, notwithstanding the presence of ubiquitous singularities stemming from the very foundations of the theory, that is the causal structure of (free) propagators. We have thus grown accustomed to the idea that the parameters in a Lagrangian have no direct physical meaning and infinite contributions may be subtracted to make sense of mathematically diverging integrals. The modern approach to renormalization [11] views the occurrence of such infinities as a measure of our theoretical ignorance of nature and every Lagrangian should, in turn, be considered as an effective (low energy) description doomed to fail at some UV energy scale Λ [12]. Moreover, gravitational corrections to the Standard Model amplitudes to a given order in the (inverse of the) Planck mass m_p are negligibly small at experimentally accessible energies [13]. These facts briefly elucidate the main theoretical reason that makes it so difficult to use gravity as a regulator: if it is to provide a natural solution to the problem of UV divergences, gravity must be treated non-perturbatively [7].

In the QFT community, gravity is mainly viewed as a spin-2 field which also happens to describe distances and angles (to some extent). As such, the most advanced strategy to deal with it is the background field method for functional integrals [14, 9], according to which one expands the Einstein-Hilbert Lagrangian (or a generalisation thereof) around all of the fields’ classical values, including the classical background metric. The latter is reserved the role of defining the causal structure of space-time, whereas the quantum mechanical part yields the

graviton propagator and matter couplings (of order m_p^{-2}). The effect of gravity on matter fields can then be analysed perturbatively by computing the relevant Feynman graphs [15]. A notorious consequence of this approach is that, by simple power counting, pure gravity is seen to be non-renormalizable, a “text-book” statement [16], yet occasionally debated. For example, Ref. [7] suggested that perturbative expansions are usually performed in the wrong variables and that Einstein gravity would appear manifestly renormalizable if one were able to resum logarithmic-like series [5]. In the physically more interesting case with matter, non-perturbative results can be obtained in just a very few cases, one of particular interest being the correction to the self-mass of a scalar particle, which becomes finite once all ladder-like graphs containing gravitons are added [4]. A remarkable approach was developed in Refs. [17], in which a tree-level effective action for gravity at the energy scale μ is derived within the background field method but without specifying the background metric *a priori*. The latter is instead, *a posteriori* and self-consistently, equated to the quantum expectation value determined by the effective action at that scale. This method does not involve cumbersome loop contributions and hints that gravity might be *non-perturbatively* renormalizable [18], with a non-Gaussian UV fixed point, thus realising the *asymptotic safety* conjectured several decades ago by Weinberg [19].

Based on the idea that QFT is an effective approach [12], different attempts have taken a shortcut and addressed the effects of gravity on the propagation of matter field directly, *e.g.*, by employing modified dispersion relations or uncertainty principles at very high (*trans-Planckian*) energy [20]. Some works have postulated such modifications, whereas others tried to derive them from (effective) descriptions of quantum gravity (see, *e.g.*, Refs. [21]). It is in fact common wisdom that, for energies of the order of m_p or larger, the machinery of QFT fails and one will need a more fundamental quantum theory of gravity, such as String Theory [22] or Loop Quantum Gravity [23]. Quite interestingly, both approaches hint at space-time non-commutativity [24] as an effective implementation of gravity as a regulator, with the scale of non-commutativity of the order of the Planck length ℓ_p . A new feature which, in turn, follows from space-time non-commutativity is the IR/UV mixing, whereby physics in the infrared (IR) is affected by UV quantities [25]. This feature gives us hope of probing (indirectly) such an extreme energy realm in future experiments or even using available data of very large scale (cosmological) structures.

In Ref. [26], we proposed yet a different strategy to incorporate gravity in the body of QFT. Instead of proposing a new, or relying on an available, fundamental theory of quantum gravity, we tried to define modified propagators in a very “conservative” (minimal) way inspired by the simple semiclassical perspective in which gravity is described by Einstein’s geometrical theory and matter by perturbative QFT. In this approach, gravity is therefore not viewed as a spin-2 field, but rather as the causal structure of space-time (or the manifestation thereof) at all loops in QFT, a property the background field method instead reserves to the classical part of the metric only. The modified propagators for matter fields should therefore take into consideration the presence of each and every source, classical or virtual, in a given process mathematically described by Feynman’s diagrams. Of course, philosophical perspectives aside, the relevant question is whether this idea leads to different (or the same) phenomenological predictions with respect to the other approaches to UV physics currently available. However, we are in a fairly premature stage to assess that. In fact, even realising the relatively simple guidelines which we review here poses serious technical problems, and a very preliminary attempt, based on several further working assumptions, can also be found in the second part of Ref. [26].

We shall use units with $c = \hbar = 1$ and the Newton constant $G = \ell_p/m_p$.

2. Geometrical gravity in QFT

In order to make contact with the physics, let us note that one needs to consider two basic energy scales, one related to phenomenology and one of theoretical origin, namely:

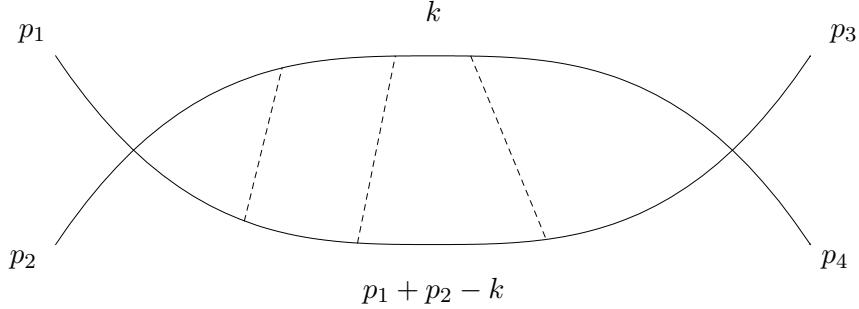


Figure 1. Usual one-loop correction to the four-point function in $\lambda \phi^4$ (solid lines) and graviton exchanges (dashed lines).

- a) the highest energy presently available in experiments, say $E_{\text{exp}} \simeq 1 \text{ TeV}$, and
- b) the Planck energy $m_p \simeq 10^{16} \text{ TeV}$.

It is well assessed that, for energies up to E_{exp} , the Standard Model of particle physics (without gravity) and renormalization techniques yield results in very good agreement with the data. Further, finite, albeit experimentally negligible, quantum gravitational corrections can be obtained by employing the effective QFT approach [13] (which also yields some – but not all – of the general relativistic corrections to the Newtonian potential). At the opposite end of the spectrum, for energies of the order of m_p or larger, QFT presumably breaks down and one needs a new quantum theory which includes gravity in a fundamental manner, like String Theory [22] or Loop Quantum Gravity [23].

In any case, we expect that gravitational corrections to QFT amplitudes play an increasingly important role for larger and larger energy scale $\mu > E_{\text{exp}}$, and that it should be possible to describe such effects in perturbative QFT directly (at least in the regime $E_{\text{exp}} \lesssim \mu \lesssim m_p$). We call this window the realm of “semiclassical gravity”, and that is the range where our proposal is more likely to shed some new light ¹.

2.1. Semiclassical gravity

At intermediate energies $E_{\text{exp}} \lesssim \mu \ll m_p$, we expect that a semiclassical picture holds in which the space-time can be reliably described as a classical manifold with a metric tensor $g_{\alpha\beta}$ that responds to the presence of (quantum) matter sources according to the well-known equation [10]

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = \frac{\ell_p}{m_p} \langle \hat{T}_{\alpha\beta} \rangle , \quad (1)$$

where $R_{\alpha\beta}$ (R) is the Ricci tensor (scalar) and $\langle \hat{T}_{\alpha\beta} \rangle$ the expectation value of the matter stress tensor obtained from QFT on that background. All the same, if one takes Eq. (1) at face value, the way perturbative terms are computed in QFT appears questionable, since loops of virtual particles are included in Feynman’s diagrams whose four-momentum $|k^2| = |k_\alpha k^\alpha|$ formally goes all the way to infinity (*i.e.*, to m_p and beyond) but are still described by the (free) propagators computed on a fixed (possibly flat) background.

For example, let us consider the graph for scalar particles with self-interaction $\lambda \phi^4$, represented by solid lines in Fig. 1, which is a pictorial representation of the integral

$$\Gamma^{(4)}(p) \simeq \int \frac{k^3 dk}{(2\pi)^4} \tilde{G}_F(k) \tilde{G}_F(p-k) , \quad (2)$$

¹ We actually attempted at pushing our predictions even further and address the very problem of UV divergences in Ref. [26].

where \tilde{G}_F is the momentum-space Feynman propagator in four dimensions,

$$\tilde{G}_F(p) = \frac{1}{p^2 + i\epsilon} . \quad (3)$$

Although the external momenta p_i ($i = 1, \dots, 4$) are taken within the range of experiments (that is, $|p_i^2| \lesssim E_{\text{exp}}^2$ in the laboratory frame), the two virtual particles in the loop have unconstrained momenta k and $p_1 + p_2 - k$, respectively. One might therefore wonder if it is at all consistent to describe those two particles using the above flat-space propagator. The common QFT approach to this problem would result in adding gravity in the form of graviton exchanges (the dashed lines in Fig. 1) and estimate deviations from purely flat-space results. This procedure is however likely to miss non-perturbative contributions that the UV physics might induce into the IR. For sure, it will not render finite diverging integrals, such as the one in Eq. (2), unless one is able to resum an infinite number of perturbative terms.

The interplay among propagators, UV divergences and the causal structure of space-time can be better appreciated by noting that, in any approach in which the space-time structure is a fixed background, the short distance behaviour of QFT (in four dimensions) is described by the Hadamard form of the propagators [6]

$$G(x, x') = \frac{U(x, x')}{\sigma} + V(x, x') \ln(\sigma) + W(x, x') , \quad (4)$$

where U , V and W are regular functions and 2σ is the square of the geodesic distance between x and x' . For instance, in Minkowski space-time, one has

$$2\sigma = (x - x')^2 , \quad (5)$$

and the propagator contains divergences for $\sigma \rightarrow 0$ (*i.e.*, along the light cone and for $x \rightarrow x'$). Calculations based on the use of propagators in QFT therefore (implicitly) rely on the formalism of distribution theory and UV divergences appear as a consequence when one tries to compute (mathematically) ill-defined quantities, such as the four-point function in Eq. (2). One can devise mathematical workarounds for this problem, but what matters here is that, if only the relation (5) is modified (like in QFT on a curved space-time), the divergences for $\sigma \rightarrow 0$ will remain. Nonetheless, a few partial results suggest that deeper modifications of the causal structure might occur at the quantum level. For example, it was shown that the divergence on the light-cone disappears (with a smearing at large momenta of the form considered in Ref. [27]) if graviton fluctuations are in a coherent state [6]².

It seems appropriate to us to tackle this problem by pushing further the validity of the semiclassical Einstein equations. We shall hence assume that virtual particles propagate in a background compatible with Eq. (1) at the scale $\mu \sim k = \sqrt{|k^2|}$ and their propagators be correspondingly adjusted [3]. As we mentioned before, our underlying viewpoint is not that gravity is just a field (although with a very complicated dynamics), but the geometrical perspective according to which gravity *is* the space-time and, in particular, the causal structure obeyed by all (other) fields. Let us remark again that this view is partly incorporated into the background field method, whereby the metric is split into two parts,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} . \quad (6)$$

The classical part $\eta_{\mu\nu}$ possesses the expected symmetries of General Relativity and determines the causal structure for all (other) classical and quantum fields, whereas $h_{\mu\nu}$ is just another

² With the further inclusion of negative norm states, all UV divergences were claimed to be cured in Ref. [28].

quantum field which acts on the matter fields via usual (although complicated) interaction terms, hence in a non-geometrical way. One could actually view our approach as a step backward, since the gravitational field is not explicitly quantised [29] (there is no analogue of the above $h_{\mu\nu}$), and it is in fact not even defined separately (*i.e.*, in the absence of matter ³).

2.2. Gravity in propagators and transition amplitudes

We can now formulate the basic prescriptions for defining a “gravitationally renormalised” QFT:

- A1)** perturbative QFT defined by Feynman diagrams is a viable approach to particle physics for energies μ below a cut-off $\Lambda \gg E_{\text{exp}}$;
- A2)** in a (one-particle irreducible) Feynman diagram with N internal lines, each virtual particle is described by a Feynman propagator

$$G(x, y) = G_{\{x_i\}}^{(\Lambda)}(x, y) \quad (7)$$

corresponding to the space-time generated by the other $N - 1$ virtual particles in the same graph with coordinate positions x_i ($i = 1, \dots, N - 1$) and constrained according to **A1**;

- A3)** Standard Model results are recovered at low energy, $\mu \lesssim E_{\text{exp}} \ll m_p$.

Several comments on the above guidelines are in order. First of all, we explicitly introduced a cut-off in **A1**, having in mind that our approach is not meant to be the final theory of everything, but should rather be regarded as a computational recipe. A second, essential, simplification was introduced in **A2**, in that each virtual particle is treated like a test particle in the space-time generated by the other particles, its own gravitational backreaction thus being neglected ⁴. Another consequence of **A2** is that integration over positions inside loops can now be viewed as also purporting a (quantum mechanical) superposition of (virtual) metrics, and there is hope that this can smear the usual divergences of (3) out (as was shown in Ref. [6] for particular gravitational states). It also should not go unnoticed that we did not mention a Lagrangian (or action) from which the modified propagators satisfying **A2** could be obtained. In this respect, our proposal follows the philosophy of Ref. [15], which gives the Lagrangian a secondary role with respect to Feynman’s rules for computing perturbative amplitudes. However, the symmetries of a system are far more readable if a Lagrangian is available [12] and it would be interesting to find out whether an action principle can be devised to streamline the derivation and show which symmetries are preserved or broken. The latter kind of analysis can also be performed perturbatively, although, as is well known for the Slavnov-Taylor identities of (non-Abelian) Yang-Mills theory, that task requires a lot more effort. A final observation is that the Standard Model of particle physics (without gravity) is a rigid theory and it is very likely that a generic modification of the sort we are proposing here has hazardous effects in the range of presently available data, thus compromising **A3**. One should therefore check very carefully that none of the assessed predictions of the Standard Model is lost in our approach.

One cannot ignore the technical fact that the N -body problem in General Relativity is extremely complicated, to say the least, already for $N = 2$. To the general guidelines, we therefore add two working assumptions:

- W1)** starting from the coordinate-space propagator in Eq. (7), it is possible to define a momentum-space propagator $\tilde{G}_{\{k_i\}}(p)$;

³ This is somewhat reminiscent of the “relational mechanics” approach to gravity (see, *e.g.*, Ref. [30] and References therein).

⁴ Let us note in passing that this somewhat parallels a perturbative result of non-commutative QFT, according to which there is no tree-level correction to the commutative case [24].

W2) one can approximate the momentum-space propagator for each virtual particle

$$\tilde{G}_{\{k_i\}}^{(\Lambda)}(p) \simeq \tilde{G}_q^{(\Lambda)}(p) , \quad (8)$$

where $q \simeq \sqrt{|\sum k_i|^2}$ is the total momentum of the remaining $N - 1$ particles.

The latter is a “mean field” assumption devised to deal with graphs containing more than two virtual particles. The approximate equality in Eq. (8) may thus be replaced with other expressions of choice, the key point being that the problem reduces to studying the propagator for a test particle in a background generated by an “average” source.

For example, the “gravitationally renormalized” analogue of the four-point amplitude in Eq. (2) is obtained by replacing each particle’s propagator with the new expression (8),

$$\Gamma_{\text{GR}}^{(4)}(p; \Lambda) \simeq \int^{\Lambda} \frac{k^3 dk}{(2\pi)^4} \tilde{G}_{(p-k)}^{(\Lambda)}(k) \tilde{G}_{(k)}^{(\Lambda)}(p - k) . \quad (9)$$

Provided $\tilde{G}_{(k)}(q)$ falls off fast enough at large k , one can therefore hope to obtain finite transition amplitudes even in the limit $\Lambda \rightarrow \infty$. Of course, in order to obtain explicit expressions, one first needs to solve for the geometry produced by virtual particles and then obtain the momentum-space form of the propagator (an early attempt can be found in the second part of Ref. [26]).

3. Final remarks

Inspired by the observation that a semiclassical description of gravity should be possible in processes that involve energies below the Planck scale, we formulated general guidelines that can be employed to adjust QFT in order to include gravitational contributions. Such guidelines were listed in the form of general prescriptions (**A1-A3**) and more specific working assumptions (**W1** and **W2**) that formalise our approach to include gravity within the Standard Model of particle physics in a geometrical and non-perturbative way. All of them are of course debatable and subject to possible refinements.

Preliminary results, reported in Ref. [26], showed that one might indeed expect significant UV modifications from the dependence of the propagators on the momenta of virtual particles. However, further working assumptions were used therein, whose impact must be clarified. And, of course, a realistic QFT should be analysed before the final word can be spoken on UV divergences and that old idea of Pauli.

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